

# Unique heavy lepton signature at $e^+e^-$ linear collider with polarized beams

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## Abstract

We explore the effects of neutrino and electron mixing with exotic heavy leptons in the process  $e^+e^- \rightarrow W^+W^-$  within  $E_6$  models. We examine the possibility of uniquely distinguishing and identifying such effects of heavy neutral lepton exchange from  $Z$ - $Z'$  mixing within the same class of models and also from analogous ones due to competitor models with anomalous trilinear gauge couplings (AGC) that can lead to very similar experimental signatures at the  $e^+e^-$  International Linear Collider (ILC) for  $\sqrt{s} = 350, 500$  GeV and 1 TeV. Such clear identification of the model is possible by using a certain double polarization asymmetry. The availability of both beams being polarized plays a crucial role in identifying such exotic-lepton admixture. In addition, the sensitivity of the ILC for probing exotic-lepton admixture is substantially enhanced when the polarization of the produced  $W^\pm$  bosons is considered.

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## I. INTRODUCTION

Detailed examination of the process

$$e^+ + e^- \rightarrow W^+ + W^- \quad (1)$$

at the ILC is a crucial one for studying the electroweak gauge symmetry, in particular, electroweak symmetry breaking and the structure of the gauge sector in general, and allows to observe a manifestation of New Physics (NP) that may appear beyond the Standard Model (SM). In the SM, the process (1) is described by the amplitudes mediated by photon and  $Z$  boson exchange in the  $s$ -channel and by neutrino exchange in the  $t$ -channel. This reaction is quite sensitive to both the leptonic vertices and the trilinear couplings to  $W^+W^-$  of the SM  $Z$  and of any new heavy neutral boson or a new heavy lepton that can be exchanged in the  $s$ -channel or  $t$ -channel, respectively. A popular example in this regard, is represented by  $E_6$  models [1–6]. In particular, an effective  $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$  model, which originates from the breaking of the exceptional group  $E_6$ , leads to extra gauge bosons. Indeed, in the breaking of this group down to the SM symmetry, two additional neutral gauge bosons could appear and the lightest  $Z'$  is defined as

$$Z' = Z'_\chi \cos \beta + Z'_\psi \sin \beta \quad (2)$$

and can be parametrized in terms of the hypercharges of the two groups  $U(1)_\psi$  and  $U(1)_\chi$  which are involved in the breaking of the  $E_6$  group into a low-energy group of rank 6:

$$\begin{aligned} E_6 &\rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi \\ &\rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_\psi \times U(1)_\chi. \end{aligned} \quad (3)$$

For a sufficiently large vacuum expectation value of the Higgs field an effective rank-5 model, which leads to the decomposition (see, for example Ref. [7])  $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{Y'}$  can be deduced from the rank-6 model (see below) so that one of the new gauge bosons decouples from low energy phenomenology. The remaining (lighter) new gauge bosons  $Z'$  is in general a mixture of  $Z_\psi$  and  $Z_\chi$  and is assumed to lead to measurable effects at the collider, and an angle  $\beta$  specifies the orientation of the  $U(1)'$  generator in the  $E_6$  group space, where the values  $\beta = 0$  and  $\beta = \pi/2$  would correspond, respectively, to pure  $Z'_\chi$  and  $Z'_\psi$  bosons, while the value  $\beta = -\arctan \sqrt{5/3}$  would correspond to a  $Z'_\eta$  boson originating from the direct breaking of  $E_6$  to a rank-5 group in superstring inspired models.

Another characteristic of extended models, apart from the  $Z'$ , is the existence of new matter, new heavy leptons and quarks. In  $E_6$  models the fermion sector is enlarged, since the matter multiplets are in larger representations (the  $\underline{27}$  fundamental representation), that contains, in particular, a vector doublet of leptons. From the phenomenological point of view it is convenient to classify the fermions present in  $E_6$  in terms of their transformation properties under  $SU(2)$ . We denote the particles with unconventional isospin assignments (right-handed doublets) as exotic fermions. We here consider two heavy left- and right-handed  $SU(2)$  exotic lepton doublets [8, 9]

$$\begin{pmatrix} N \\ E^- \end{pmatrix}_L, \quad \begin{pmatrix} N \\ E^- \end{pmatrix}_R, \quad (4)$$

and one  $Z'$  boson, with masses larger than  $M_Z$  and coupling constants that may be different from those of the SM. These leptons are called *vector leptons* because both the left- and right-handed components transform identically under  $SU(2)$ . We also assume that the new, “exotic” fermions only mix with the standard ones within the same family (the electron and its neutrino being the ones relevant to process (1)), which assures the absence of tree-level generation-changing neutral currents [10].

Current lower limits on  $M_{Z'}$  obtained from dilepton pair production at the LHC with  $\sqrt{s} = 8$  TeV and  $\mathcal{L}_{\text{int}} \approx 20 \text{ fb}^{-1}$  [11, 12] range in the interval  $\sim 2.6 - 2.9$  TeV, depending on the particular  $Z'$  model being tested. Already these masses are too high for a  $Z'$  to be directly seen at the ILC. However, even at such high masses,  $Z'$  exchanges can manifest themselves indirectly *via* deviations of cross sections, and in general of the reaction observables, from the SM predictions.

In this paper, we study the indirect effects induced by heavy lepton exchange in  $W^\pm$  pair production (1) at the ILC, with a center of mass energy  $\sqrt{s} = 0.5 - 1$  TeV and time-integrated luminosity of  $\mathcal{L}_{\text{int}} = 0.5 - 1 \text{ ab}^{-1}$ . We also present results for a lower energy run at  $\sqrt{s} = 350$  GeV. For early papers on these effects, see Refs. [13–15]. We allow for effects due to extra  $Z'$  gauge boson exchange. Indirect effects may be quite subtle, both when it comes to distinguishing an effect from the SM, and also as far as the identification of the source of an observed deviation is concerned, because *a priori* different NP scenarios may lead to the same or similar experimental signatures. Clearly, then, the discrimination of one NP model (in our case the  $E_6$ ) from other possible ones needs an appropriate strategy for analyzing the data.

Recently, the problem of distinguishing the  $Z'$  effects, once observed in process (1), from the anomalous gauge couplings, has been studied in [16]. In the AGC models, there is no new gauge boson exchange, but the  $WW\gamma$ ,  $WWZ$  couplings are modified with respect to the SM values, this violates the SM gauge cancellation too and leads to deviations of the cross sections. We consider the CP-conserving set of such couplings, often referred to as  $\kappa_\gamma$ ,  $\kappa_Z$ ,  $\lambda_\gamma$ ,  $\lambda_Z$  and  $\delta_Z$  [17, 18]. An alternative effective-field-theory approach to these effects was recently presented [19].

In this note, we extend the analysis of Ref. [16], considering the possibility of uniquely identifying the effects of heavy neutral lepton exchange from  $Z$ - $Z'$  mixing within the same class of  $E_6$  models. This is relevant, since in this class of models lepton mixing and  $Z$ - $Z'$  mixing can be simultaneously present. We also distinguish them from analogous ones due to competitor models with anomalous trilinear gauge couplings in the process (1) by exploiting a double polarization asymmetry that will unambiguously identify the heavy exotic-lepton mixing effects<sup>1</sup> and is only accessible with the availability of both beams being polarized [21].

While the high precision observables determined at LEP severely constrain the electroweak sector [22], they leave room for effects at the energies that are discussed here.

The paper is organized as follows. In Section II, we briefly review the  $E_6$  models involving additional  $Z'$  bosons and new heavy charged and neutral leptons and emphasize the role of the heavy neutral lepton and boson mixings in the process (1). Then, in Sect. III we review the structure of the polarized cross section. In Sect. IV we determine the discovery reach on the  $NWe$  coupling constants, and in Sect. V we determine the identification reach, i.e., down to what coupling strength such a heavy neutral lepton can be distinguished from other new-physics effects. Then, in Sect. VI we comment on the 350 GeV option, before concluding in Sect. VII.

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<sup>1</sup> This approach was recently exploited for uniquely identifying the indirect effects of  $s$ -channel sneutrino exchange against other new physics scenarios described by contact-like effective interactions in high-energy  $e^+e^-$  annihilation into lepton pairs [20].

## II. LEPTON AND $Z - Z'$ MIXING

### A. Weak basis

To describe the formalism for mixing among exotic and ordinary leptons, we start from the leptonic  $SU(2) \times U(1) \times U(1)'$  interaction:<sup>2</sup>

$$-\mathcal{L} = e \left( \tilde{J}_{\text{em}}^\mu A_\mu + \tilde{J}_Z^\mu Z_\mu + \tilde{J}_{Z'}^\mu Z'_\mu \right) + \frac{g}{\sqrt{2}} \left( \tilde{J}_W^\mu W_\mu + \text{h.c.} \right), \quad (5)$$

where, in the weak-eigenstate basis, and with  $V = \gamma, Z, Z'$ , the currents in Eq. (5) can be written as:

$$\tilde{J}_V^\mu = \sum_a \bar{\varepsilon}_a^0 \gamma^\mu Q_a^{\varepsilon^0} \varepsilon_a^0, \quad \tilde{J}_W^\mu = \sum_a \bar{\eta}_a^0 \gamma^\mu G_a^{\eta^0} \varepsilon_a^0, \quad (6)$$

where the coupling matrices  $Q_a^{\varepsilon^0}$  and  $G_a^{\eta^0}$  of the neutral and charged currents are defined by Eqs. (8) and (11) below. The superscript “0” labels the weak-eigenstate basis. Furthermore, in Eq. (5) we adopt the following notations:  $e = \sqrt{4\pi\alpha_{\text{em}}}$ ,  $g = e/s_W$ ,  $s_W = \sin\theta_W$ . In Eq. (6), we have introduced, with  $a = (L, R)$  the left- and right-handed helicities, the charged and neutral leptons by means of the notation:

$$\varepsilon_a^0 = \begin{pmatrix} e_a^0 \\ E_a^0 \end{pmatrix}, \quad \eta_a^0 = \begin{pmatrix} \nu_a^0 \\ N_a^0 \end{pmatrix}, \quad (7)$$

where  $e$  and  $\nu$  are the ordinary SM electron and neutrino, and  $E$  and  $N$  are the exotic charged and neutral heavy leptons, which we assume to be doublets under electroweak  $SU(2)$ . Furthermore, the neutral current couplings are represented by the matrices  $Q_a^{\varepsilon^0} = Q_{\text{em},a}^{\varepsilon^0}; g_a^{\varepsilon^0}; g_a'^{\varepsilon^0}$ , with:

$$Q_{\text{em},a}^{\varepsilon^0} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad g_a^{\varepsilon^0} = \begin{pmatrix} g_a^{e^0} & 0 \\ 0 & g_a^{E^0} \end{pmatrix}, \quad g_a'^{\varepsilon^0} = \begin{pmatrix} g_a'^{e^0} & 0 \\ 0 & g_a'^{E^0} \end{pmatrix}, \quad (8)$$

for the  $\gamma, Z$  and  $Z'$ , respectively, where  $(\varepsilon^0 = e^0, E^0)$

$$g_a^{\varepsilon^0} = (T_{3a}^{\varepsilon^0} - Q_{\text{em},a}^{\varepsilon^0} s_W^2) g_Z, \quad (9)$$

and  $T_{3a}^{\varepsilon^0}$  is the third isospin component. Furthermore,  $g_Z = 1/s_W c_W$ , with  $c_W = \cos\theta_W$ .

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<sup>2</sup> The needed fermion mixing formalism has been introduced also, e.g., in [15].

For the  $Z'$  couplings to fermions in  $E_6$  models, we follow the notation of [15]:

$$\begin{aligned} g_L^{e^0} &= (3A + B)g_{Z'}, & g_R^{e^0} &= (A - B)g_{Z'}, \\ g_L^{E^0} &= (-2A - 2B)g_{Z'}, & g_R^{E^0} &= (-2A + 2B)g_{Z'}, \end{aligned} \quad (10)$$

where  $g_{Z'} = 1/c_W$ ,  $A = \cos \beta / (2\sqrt{6})$ ,  $B = \sqrt{10} \sin \beta / 12$ .

The charged current couplings read:

$$G_a^{\eta^0} = \begin{pmatrix} G_a^{\nu^0} & 0 \\ 0 & G_a^{N^0} \end{pmatrix} \quad (11)$$

with  $G_L^{\nu^0} = 1$ ,  $G_R^{\nu^0} = 0$ ,  $G_a^{N^0} = -2T_{3a}^E$ .

## B. Fermion mass basis

We introduce mass eigenstates in the same notation as (7):

$$\varepsilon_a = \begin{pmatrix} e_a \\ E_a \end{pmatrix}, \quad \eta_a = \begin{pmatrix} \nu_a \\ N_a \end{pmatrix}. \quad (12)$$

These states are related to the weak eigenstates (7) by the following transformations:

$$\varepsilon_a = U(\psi_{1a})\varepsilon_a^0; \quad \eta_a = U(\psi_{2a})\eta_a^0, \quad (13)$$

where the unitary mixing matrices  $U(\psi_{1a})$  and  $U(\psi_{2a})$  diagonalize, respectively, the charged and neutral fermion mass matrices.  $U(\psi_{1a})$  and  $U(\psi_{2a})$  can be written as:

$$U(\psi_{1a}) = \begin{pmatrix} \cos \psi_{1a} & \sin \psi_{1a} \\ -\sin \psi_{1a} & \cos \psi_{1a} \end{pmatrix} \equiv \begin{pmatrix} c_{1a} & s_{1a} \\ -s_{1a} & c_{1a} \end{pmatrix}, \quad (14)$$

$$U(\psi_{2a}) = \begin{pmatrix} \cos \psi_{2a} & \sin \psi_{2a} \\ -\sin \psi_{2a} & \cos \psi_{2a} \end{pmatrix} \equiv \begin{pmatrix} c_{2a} & s_{2a} \\ -s_{2a} & c_{2a} \end{pmatrix}. \quad (15)$$

Present limits on  $s_{1a}^2$  and  $s_{2a}^2$  are in general less than 1-2% [9, 23, 24] and  $m_N > 100$  GeV [10]. In the fermion-mass-eigenstate basis one can rewrite the interaction Lagrangian (5) as:

$$-\mathcal{L} = e \left( J_{\text{em}}^\mu A_\mu + J_Z^\mu Z_\mu + J_{Z'}^\mu Z'_\mu \right) + \frac{g}{\sqrt{2}} (J_W^\mu W_\mu + \text{h.c.}), \quad (16)$$

where

$$J_V^\mu = \sum_a \bar{\varepsilon}_a \gamma^\mu Q_a^\varepsilon \varepsilon_a, \quad J_W^\mu = \sum_a \bar{\eta}_a \gamma^\mu G_a^\eta \varepsilon_a. \quad (17)$$

Since the gauge fields of Eq. (16) are the same as those of (5), we must have

$$Q_a^\varepsilon = U(\psi_{1a}) Q_a^{\varepsilon^0} U^{-1}(\psi_{1a}), \quad G_a^\eta = U(\psi_{2a}) G_a^{\eta^0} U^{-1}(\psi_{1a}), \quad (18)$$

and  $Q_a^\varepsilon = Q_{em,a}^\varepsilon, g_a^\varepsilon, g_a^{\prime\varepsilon}$ , with

$$g_a^\varepsilon = \begin{pmatrix} g_a^e & g_a^{eE} \\ g_a^{eE} & g_a^E \end{pmatrix}, \quad g_a^{\prime\varepsilon} = \begin{pmatrix} g_a^{\prime e} & g_a^{\prime eE} \\ g_a^{\prime eE} & g_a^{\prime E} \end{pmatrix}, \quad G_a^\eta = \begin{pmatrix} G_a^\nu & G_a^{\nu E} \\ G_a^{Ne} & G_a^N \end{pmatrix}. \quad (19)$$

It is clear that the electromagnetic current remains diagonal under the rotation (18), and therefore is not affected by lepton mixing.

In the weak charged currents of Eq. (17) the exotic-lepton mixings modify not only the left-handed currents but also induce an admixture with the right-handed currents. The off-diagonal term in  $J_W^\mu$  of Eqs. (17)–(19) induces  $NWe$  couplings which allow an additional  $t$ -channel exotic-lepton-exchange contribution for the process (1) (see Fig. 1). Parametrization of the mixing-modified  $\nu We$  and the mixing-induced  $NWe$  couplings are summarized in Eqs. (21) and (22), respectively.

From (18) and (19) one can obtain expressions for the lepton coupling constants:

$$g_a^e = g_a^{e^0} c_{1a}^2 + g_a^{E^0} s_{1a}^2, \quad g_a^{\prime e} = g_a^{\prime e^0} c_{1a}^2 + g_a^{\prime E^0} s_{1a}^2; \quad (20)$$

$$G_L^\nu = c_{1L} c_{2L} - 2 T_{3L}^E s_{1L} s_{2L}, \quad G_R^\nu = -2 T_{3R}^E s_{1R} s_{2R}; \quad (21)$$

$$G_L^{Ne} = -s_{2L} c_{1L} - 2 T_{3L}^E c_{2L} s_{1L}, \quad G_R^{Ne} = -2 T_{3R}^E c_{2R} s_{1R}. \quad (22)$$

### C. $Z$ - $Z'$ mixing

Concerning  $Z$ - $Z'$  mixing, it can be parametrized as

$$\begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} Z \\ Z' \end{pmatrix}, \quad (23)$$

where  $Z, Z'$  are weak eigenstates,  $Z_1, Z_2$  are mass eigenstates and  $\phi$  is the  $Z$ - $Z'$  mixing angle. Finally, taking Eq. (23) into account, the lepton neutral current couplings to  $Z_1$  and  $Z_2$  are, respectively [15]:

$$g_{1a}^e = g_a^e \cos \phi + g_a^{\prime e} \sin \phi; \quad g_{2a}^e = -g_a^e \sin \phi + g_a^{\prime e} \cos \phi. \quad (24)$$

Current limits are of the order  $\phi = (2 - 5) \times 10^{-3}$  [10].

### III. POLARIZED CROSS SECTION

In the Born approximation the process (1) is described by the set of five diagrams shown in Fig. 1 and corresponding to mass-eigenstate exchanges (i.e.  $\gamma$ ,  $\nu$ ,  $N$ ,  $Z_1$  and  $Z_2$ ), with couplings given by Eqs. (20)-(22) and (24).

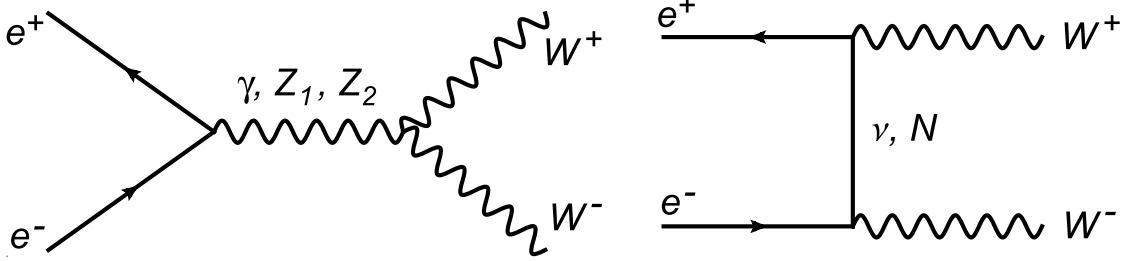


FIG. 1: Feynman diagrams.

The polarized cross section for the process (1) can be written as [15]

$$\begin{aligned} \frac{d\sigma(P_L^-, P_L^+)}{d\cos\theta} = \frac{1}{4} \left[ (1 + P_L^-)(1 - P_L^+) \frac{d\sigma^{RL}}{d\cos\theta} + (1 - P_L^-)(1 + P_L^+) \frac{d\sigma^{LR}}{d\cos\theta} \right. \\ \left. + (1 + P_L^-)(1 + P_L^+) \frac{d\sigma^{RR}}{d\cos\theta} + (1 - P_L^-)(1 - P_L^+) \frac{d\sigma^{LL}}{d\cos\theta} \right], \quad (25) \end{aligned}$$

where  $P_L^-$  ( $P_L^+$ ) are degrees of longitudinal polarization of  $e^-$  ( $e^+$ ),  $\theta$  the scattering angle of the  $W^-$  with respect to the  $e^-$  direction. The superscript “RL” refers to a right-handed electron and a left-handed positron, and similarly for the other terms. The relevant polarized differential cross sections for  $e_a^- e_b^+ \rightarrow W_\alpha^- W_\beta^+$  contained in Eq. (25) can be expressed as [15, 25]

$$\frac{d\sigma_{\alpha\beta}^{ab}}{d\cos\theta} = C \sum_{k=0}^{k=2} F_k^{ab} \mathcal{O}_{k\alpha\beta}, \quad (26)$$

where  $C = \pi\alpha_{e.m.}^2\beta_W/2s$ ,  $\beta_W = (1 - 4M_W^2/s)^{1/2}$  the  $W$  velocity in the CM frame, and the helicities of the initial  $e^-e^+$  and final  $W^-W^+$  states are labeled as  $ab = (RL, LR, LL, RR)$  and  $\alpha\beta = (LL, TT, TL)$ , respectively. The  $\mathcal{O}_k$  are functions of the kinematical variables dependent on energy  $\sqrt{s}$ , the scattering angle  $\theta$  and the  $W$  mass,  $M_W$ , which characterize the various possibilities for the final  $W^+W^-$  polarizations ( $TT$ ,  $LL$ ,  $TL + LT$  or the sum over all  $W^+W^-$  polarization states for unpolarized  $W$ 's).

The  $F_k$  are combinations of lepton and trilinear gauge boson couplings,  $g_{WWZ_1}$  and  $g_{WWZ_2}$ , including lepton and  $Z$ - $Z'$  mixing as well as propagators of the intermediate states.



For instance, for the  $LR$  case one finds

$$\begin{aligned} F_0^{LR} &= \frac{1}{16s_W^4} \left[ (G_L^\nu)^2 + r_N (G_L^{Ne})^2 \right]^2, \\ F_1^{LR} &= 2 \left[ 1 - g_{WWZ_1} g_{1L}^e \chi_1 - g_{WWZ_2} g_{2L}^e \chi_2 \right]^2, \\ F_2^{LR} &= -\frac{1}{2s_W^2} \left[ (G_L^\nu)^2 + r_N (G_L^{Ne})^2 \right] \left[ 1 - g_{WWZ_1} g_{1L}^e \chi_1 - g_{WWZ_2} g_{2L}^e \chi_2 \right], \end{aligned} \quad (27)$$

where the  $\chi_j$  ( $j = 1, 2$ ) are the  $Z_1$  and  $Z_2$  propagators, i.e.  $\chi_j = s/(s - M_j^2 + iM_j\Gamma_j)$ ,  $r_N = t/(t - m_N^2)$ , with  $t = M_W^2 - s/2 + s \cos \theta \beta_W/2$ , and  $m_N$  is the neutral heavy lepton mass. Also, in Eq. (27),  $g_{WWZ_1} = g_{WWZ} \cos \phi$  and  $g_{WWZ_2} = -g_{WWZ} \sin \phi$  where  $g_{WWZ} = \cot \theta_W$ . Note that Eq. (27) is obtained in the approximation where the imaginary parts of the  $Z_1$  and  $Z_2$  boson propagators are neglected, which is fully appropriate far away from the poles. (Accounting for this effect would require the replacements  $\chi_j \rightarrow \text{Re } \chi_j$  and  $\chi_j^2 \rightarrow |\chi_j|^2$  on the right-hand side of Eq. (27).)

Since the gauge eigenstate  $Z'$  is neutral under  $SU(2)_L$  and does not couple to the  $W^+W^-$  pair, the process (1) is sensitive to a  $Z'$  only in the case of a non-zero  $Z$ - $Z'$  mixing. Moreover, as one can easily see from the formulae above, the  $s$ -channel  $Z_2$  and the  $t$ -channel  $N$  exchange amplitudes arise only in the case of non-vanishing mixing angles. In this case, the expression for the SM cross section [25] can be obtained from (25) in the limit of vanishing mixing angles.

The first term  $F_0^{LR}$  describes the contributions to the cross section caused by neutrino  $\nu$  and heavy neutral lepton  $N$  exchanges in the  $t$ -channel while the second one,  $F_1^{LR}$ , is responsible for  $s$ -channel exchange of the photon  $\gamma$  and the gauge bosons  $Z_1$  and  $Z_2$ . The interference between  $s$ - and  $t$ -channel amplitudes is contained in the term  $F_2^{LR}$ . The  $RL$  case is simply obtained from Eq. (27) by exchanging  $L \rightarrow R$ .

For the  $LL$  and  $RR$  cases there is only  $N$ -exchange contribution,

$$F_0^{LL} = F_0^{RR} = \frac{1}{16s_W^4} r_N^2 (G_L^{Ne} G_R^{Ne})^2. \quad (28)$$

Concerning the  $\mathcal{O}_{k\alpha\beta}$  multiplying the expression in Eq. (28) (see Eq. (26)) their explicit expressions for polarized and unpolarized final states  $W^+W^-$  can be found in, e.g. [15].

#### IV. DISCOVERY REACH ON HEAVY LEPTON COUPLINGS

We take “discovery” of new physics to mean exclusion of the Standard Model at a given confidence level. In the following, this will be the 95% C.L.

### A. No $Z$ - $Z'$ mixing

Let us start the analysis with a case where there is only lepton mixing and no  $Z$ - $Z'$  mixing, i.e.,  $\phi = 0$ . Since the mixing angles are bounded by  $s_i^2$  at most of order  $10^{-2}$ , we can expect that retaining only the terms of order  $s_1^2$ ,  $s_2^2$  and  $s_1 s_2$  in the cross section (25) should be an adequate approximation. To do that we expand the couplings of Eqs. (20)-(22) taking Eq. (9) into account. We find for  $E_6$  models, where  $T_{3L}^E = T_{3R}^E = -1/2$ :

$$\begin{aligned} G_L^{Ne} &= s_{1L} - s_{2L}, & G_R^{Ne} &= s_{1R} \\ g_L^e &= g_L^{e^0}, & g_R^e &= g_R^{e^0} - \frac{1}{2}(G_R^{Ne})^2 g_Z, \\ G_L^\nu &= G_L^{\nu^0} - \frac{1}{2}(G_L^{Ne})^2, & G_R^\nu &= s_{1R} s_{2R}. \end{aligned} \quad (29)$$

From Eqs. (27)-(29) one can see that in the adopted approximation the cross section (25) allows to constrain basically the pair of heavy lepton couplings squared,  $((G_L^{Ne})^2, (G_R^{Ne})^2)$ , it is not possible to constrain  $s_{2R}^2$ , which represents mixing in the right-handed neutral-lepton sector.

The sensitivity of the polarized differential cross section (25) to the couplings  $G_L^{Ne}$  and  $G_R^{Ne}$  is evaluated numerically by dividing the angular range  $|\cos\theta| \leq 0.98$  into 10 equal bins, and defining a  $\chi^2$  function in terms of the expected number of events  $N(i)$  in each bin for a given combination of beam polarizations [16]:

$$\chi^2 = \sum_{\{P_L^-, P_L^+\}} \sum_i^{\text{bins}} \left[ \frac{N_{\text{SM+NP}}(i) - N_{\text{SM}}(i)}{\delta N_{\text{SM}}(i)} \right]^2, \quad (30)$$

where  $N(i) = \mathcal{L}_{\text{int}} \sigma_i \varepsilon_W$  with  $\mathcal{L}_{\text{int}}$  the time-integrated luminosity. Furthermore,

$$\sigma_i = \sigma(z_i, z_{i+1}) = \int_{z_i}^{z_{i+1}} \left( \frac{d\sigma}{dz} \right) dz, \quad (31)$$

where  $z = \cos\theta$  and polarization indices have been suppressed. Also,  $\varepsilon_W$  is the efficiency for  $W^+W^-$  reconstruction, for which we take the channel of lepton pairs ( $e\nu + \mu\nu$ ) plus two hadronic jets, giving  $\varepsilon_W \simeq 0.3$  basically from the relevant branching ratios. The procedure outlined above is followed to evaluate both  $N_{\text{SM}}(i)$  and  $N_{\text{SM+NP}}(i)$ .

The uncertainty on the number of events  $\delta N_{\text{SM}}(i)$  combines both statistical and systematic errors where the statistical component is determined by  $\delta N_{\text{SM}}^{\text{stat}}(i) = \sqrt{N_{\text{SM}}(i)}$ . Concerning systematic uncertainties, an important source is represented by the uncertainty on

beam polarizations, for which we assume  $\delta P_L^-/P_L^- = \delta P_L^+/P_L^+ = 0.5\%$  with the “standard” envisaged values  $|P_L^-| = 0.8$  and  $|P_L^+| = 0.6$  [21]. As for the time-integrated luminosity, for simplicity we assume it to be equally distributed between the different polarization configurations. Another source of systematic uncertainty originates from the efficiency of reconstruction of  $W^\pm$  pairs which we assume to be  $\delta\varepsilon_W/\varepsilon_W = 0.5\%$ . Also, in our numerical analysis to evaluate the sensitivity of the differential distribution to model parameters we include initial-state QED corrections to on-shell  $W^\pm$  pair production in the flux function approach [26–30] that assures a good approximation within the expected accuracy of the data.

As a criterion to derive constraints on the coupling constants in the case where no deviations from the SM were observed within the foreseeable uncertainties on the measurable cross sections, we impose that

$$\chi^2 \leq \chi_{\min}^2 + \chi_{\text{CL}}^2, \quad (32)$$

where  $\chi_{\text{CL}}^2$  is a number that specifies the chosen confidence level, and  $\chi_{\min}^2$  is the minimal value of the  $\chi^2$  function.

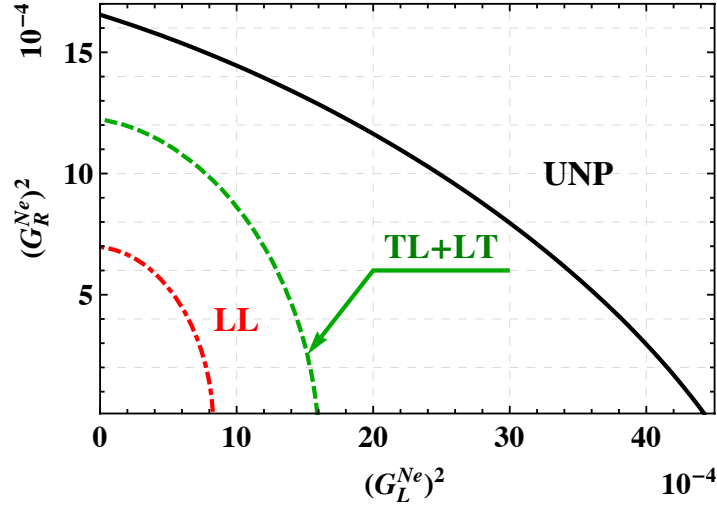


FIG. 2: Discovery reach (95% C.L.) on the heavy neutral lepton couplings  $(G_L^{Ne})^2$  and  $(G_R^{Ne})^2$  obtained from differential polarized cross sections with  $(P_L^- = \pm 0.8, P_L^+ = \mp 0.6)$  and different sets of  $W^\pm$  polarizations. Here,  $\sqrt{s} = 0.5$  TeV,  $\mathcal{L}_{\text{int}} = 0.5 \text{ ab}^{-1}$  and  $m_N = 0.3$  TeV.

From the numerical procedure outlined above, we obtain the allowed regions in  $(G_L^{Ne})^2$  and  $(G_R^{Ne})^2$  determined from the differential polarized cross sections with different sets of

polarization (as well as from the unpolarized process (1)) depicted in Fig. 2, where  $\mathcal{L}_{\text{int}} = 500 \text{ fb}^{-1}$  has been taken [21].

The results of a further potential extension of the present analysis are also shown in Fig. 2 where the feasibility of measuring polarized  $W^\pm$  states in the process (1) is assumed. This assumption is based on the experience gained at LEP2 on measurements of  $W$  polarisation [31]. The method exploited for the measurement of  $W$  polarisation is based on the spin density matrix elements that allow to obtain the differential cross sections for polarised  $W$  bosons. Information on spin density matrix elements as functions of the  $W^-$  production angle with respect to the electron beam direction was extracted from the decay angles of the charged lepton in the  $W^-$  ( $W^+$ ) rest frame. The relevant theoretical framework for measurement of  $W^\pm$  polarisation was described in [18, 25].

In Fig. 2, we consider different cases of polarized  $W$ s, with  $W_L$  and  $W_T$  referring to longitudinally and transversely polarized  $W$ s, respectively. As shown in the figure,  $d\sigma(W_L^+W_L^-)/dz$  is most sensitive to the parameters  $(G_L^{Ne})^2$  and  $(G_R^{Ne})^2$  while  $d\sigma(W_T^+W_T^-)/dz$  has the lowest sensitivity to those parameters. The bounds on heavy lepton couplings obtained from  $d\sigma(W_T^+W_T^-)/dz$  are not presented here as they are outside of the range shown in Fig. 2. The role of  $W$  polarization is seen to be essential in order to set meaningful finite bounds on the  $NWe$  couplings.

The obtained bounds are reminiscent of arcs of circles in the  $(G_L^{Ne})^2$ - $(G_R^{Ne})^2$  plane. This reflects the fact that the deviations in the  $LR$  and  $RL$  cross sections are approximately the same for the right-handed and left-handed couplings (recall that  $T_{3L}^E = T_{3R}^E$ ) and thus approximately behave as  $(G_L^{Ne})^4 + (G_R^{Ne})^4$ .

In this Fig. 2, we considered a fairly low mass,  $m_N = 0.3 \text{ TeV}$ . As one can see from Fig. 3 the constraints on heavy lepton couplings become more severe for larger values of  $m_N$ . The point is that the deviation of the cross section induced by the lepton mixing, from the SM prediction can be expressed, e.g., for the  $LR$  case, as

$$\Delta\sigma_{LR} \equiv \sigma_{\text{NP}} - \sigma_{\text{SM}} \propto (G_L^{Ne})^2 (1 - r_N), \quad (33)$$

where we have used Eqs. (27) and (29). This structure  $(1 - r_N)$  arises from negative interference between a mixing contribution to  $\nu$  exchange and the  $N$ -exchange contribution. It reflects the decreasing impact of the heavy neutrino exchange contribution to  $\Delta\sigma_{LR}$ , since at large values of  $m_N$  the last term will be small. This leads to a better sensitivity on the

mixing angles with increasing  $m_N$ . The analogous dependence also holds for  $\Delta\sigma_{RL}$  case.

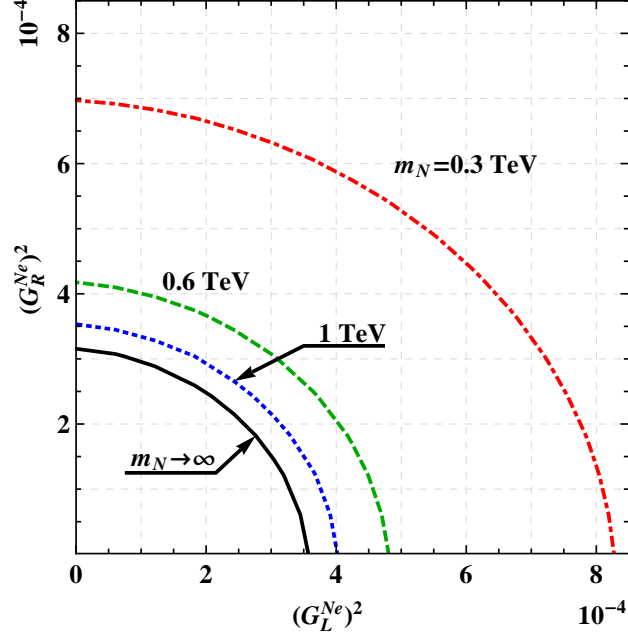


FIG. 3: Same as in Fig. 2 but obtained from the differential polarized cross sections  $d\sigma(W_L^+W_L^-)/dz$  only, with  $(P_L^- = \pm 0.8, P_L^+ = \mp 0.6)$  and different values of the lepton mass  $m_N = 0.3$  TeV, 0.6 TeV, 1 TeV and  $m_N \rightarrow \infty$ . Here,  $\sqrt{s} = 0.5$  TeV and  $\mathcal{L}_{\text{int}} = 0.5 \text{ ab}^{-1}$ .

### B. Including $Z$ - $Z'$ mixing

Now we turn to the generic case where both lepton mixing and  $Z$ - $Z'$  mixing occur, so that the leptonic coupling constants are as in Eq. (24) and the  $Z_1, Z_2$  couplings to  $W^\pm$  are as in Eq. (27). In this case, in order to evaluate the influence of the  $Z$ - $Z'$  mixing on the allowed discovery region on the heavy lepton coupling plane  $((G_L^{Ne})^2, (G_R^{Ne})^2)$  one should vary the mixing angle  $\phi$  within its current constraints which depend on the specific  $Z'$  model [32], namely  $-0.0018 < \phi < 0.0009$  for the  $\psi$  model and  $-0.0016 < \phi < 0.0006$  for the  $\chi$  model. Within a specific  $Z'$  model and with fixed  $m_N$ , the  $\chi^2$  function basically depends on three parameters:  $\phi$ ,  $G_L^{Ne}$  and  $G_R^{Ne}$ . In this case, Eq. (32) describes a tree-dimensional surface. Its projection on the  $((G_L^{Ne})^2, (G_R^{Ne})^2)$  plane demonstrates the interplay between leptonic and  $Z$ - $Z'$  mixings. Fig. 4 shows, as a typical example, the results of this analysis for the  $\chi$ -model (left panel) and the  $\psi$ -model (right panel), respectively, with fixed  $m_N = 0.3$  TeV. As one can see, the shapes of the allowed regions for the coupling constants  $G_L^{Ne}$  and  $G_R^{Ne}$

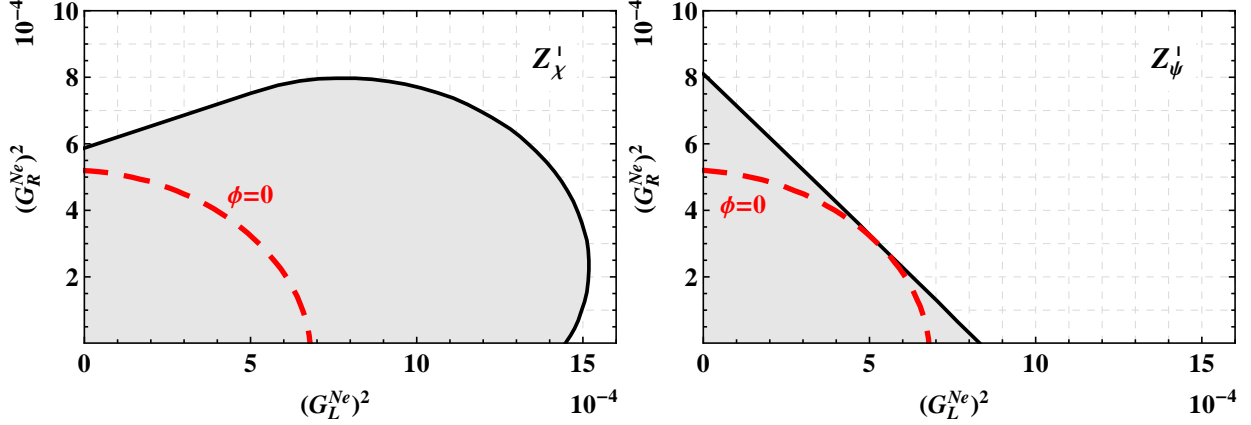


FIG. 4: Discovery reach at 95% CL on the heavy neutral lepton coupling plane  $((G_L^{Ne})^2, (G_R^{Ne})^2)$  at  $m_N = 0.3$  TeV in the case where both lepton mixing and  $Z$ - $Z'$  mixing are simultaneously allowed for the  $Z'_\chi$  model (left panel) and the  $Z'_\psi$  model (right panel), obtained from combined analysis of polarized differential cross sections  $d\sigma(W_L^+W_L^-)/dz$  at different sets of polarization,  $P_L^- = \pm 0.8$ ,  $P_L^+ = \mp 0.6$ , at the ILC with  $\sqrt{s} = 0.5$  TeV and  $\mathcal{L}_{\text{int}} = 1 \text{ ab}^{-1}$ . The dashed curves labelled “ $\phi = 0$ ” refer to the case of no  $Z$ - $Z'$  mixing.

are quite dependent on the  $Z'$  model and different for these two cases. From the explicit calculation it turns out that this is due to the different relative signs between the lepton and  $Z$ - $Z'$  mixing contributions to the deviations of the cross section  $\Delta\sigma$ .

Concerning Fig. 4 and the corresponding analysis for the  $\chi$  and  $\psi$  models, we should note that the bounds on the lepton couplings  $(G_L^{Ne})^2$  and  $(G_R^{Ne})^2$  are somewhat looser than in the case  $\phi = 0$  discussed above (roughly, by a factor as large as two), but still numerically competitive with the current situation. Also, we can remark that the cross sections for longitudinal  $W^+W^-$  production provide by themselves the most stringent constraints for this model.

Finally, one should note that although the discovery reach on the lepton couplings  $(G_L^{Ne})^2$  and  $(G_R^{Ne})^2$  obtained from polarized differential cross sections is quite dependent on the  $Z'$  model, this is not the case for the identification reach as the double beam polarization asymmetry  $A_{\text{double}}^N$  is basically independent of the  $Z$ - $Z'$  boson mixing.

## V. IDENTIFICATION OF HEAVY LEPTON EFFECTS WITH $A_{\text{double}}$

By “identification” we shall here mean *exclusion* of a certain set of competitive models, including the SM, to a certain confidence level. For this purpose, the double beam polarization asymmetry, defined as [20, 33, 34]

$$A_{\text{double}} = \frac{\sigma(P_1, -P_2) + \sigma(-P_1, P_2) - \sigma(P_1, P_2) - \sigma(-P_1, -P_2)}{\sigma(P_1, -P_2) + \sigma(-P_1, P_2) + \sigma(P_1, P_2) + \sigma(-P_1, -P_2)}, \quad (34)$$

is very useful. Here  $P_1 = |P_L^-|$ ,  $P_2 = |P_L^+|$ , and  $\sigma(\pm P_1, \pm P_2)$  denotes the polarized integrated cross section determined within the allowed range of the  $W^-$  scattering angle (or  $\cos \theta$ ). From Eqs. (25) and (34) one finds for the  $A_{\text{double}}$  of the process (1)

$$A_{\text{double}} = P_1 P_2 \frac{(\sigma^{RL} + \sigma^{LR}) - (\sigma^{RR} + \sigma^{LL})}{(\sigma^{RL} + \sigma^{LR}) + (\sigma^{RR} + \sigma^{LL})}. \quad (35)$$

We note that this asymmetry is only available if both initial beams are polarized.

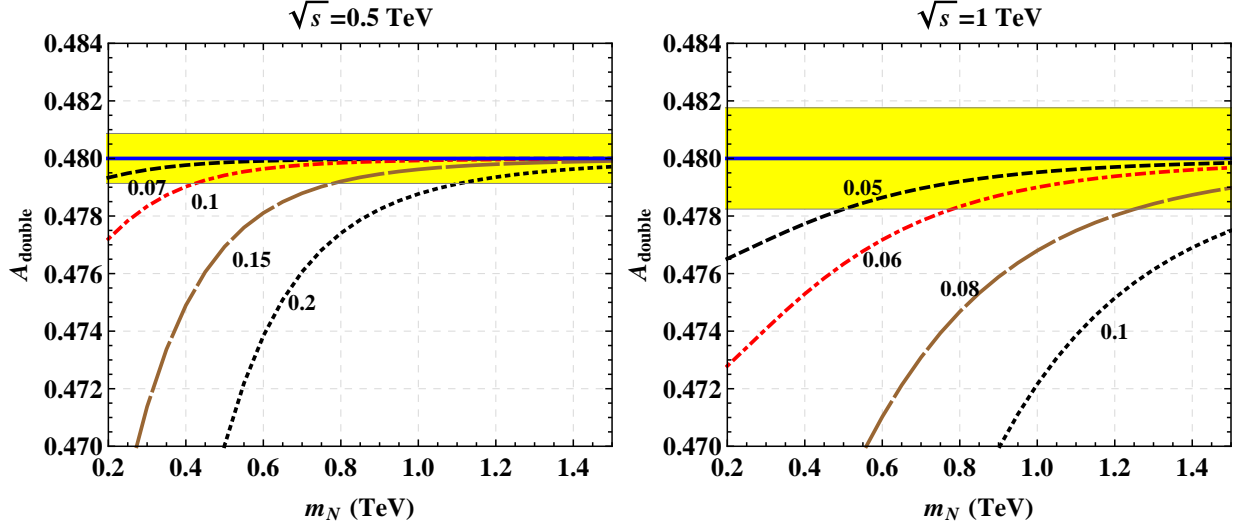


FIG. 5: Double beam polarization asymmetry  $A_{\text{double}}$  for the production of unpolarized  $W^\pm$  as a function of neutral heavy lepton mass  $m_N$  for different choices of couplings  $\sqrt{G_L^{Ne} G_R^{Ne}}$  (attached to the lines) at the ILC with  $\sqrt{s} = 0.5$  TeV (left panel) and  $\sqrt{s} = 1.0$  TeV (right panel),  $\mathcal{L}_{\text{int}} = 1 \text{ ab}^{-1}$ . The solid horizontal line corresponds to  $A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{Z'} = A_{\text{double}}^{\text{AGC}}$ . The error bands indicate the expected uncertainty in the SM case at the 1- $\sigma$  level.

It is important to also note that the SM gives rise only to  $\sigma^{LR}$  and  $\sigma^{RL}$  such that the structure of the integrated cross section has the form

$$\sigma_{\text{SM}} = \frac{1}{4} [(1 + P_L^-) (1 - P_L^+) \sigma_{\text{SM}}^{RL} + (1 - P_L^-) (1 + P_L^+) \sigma_{\text{SM}}^{LR}]. \quad (36)$$

This is also the case for anomalous gauge couplings (AGC) [25], and  $Z'$ -boson exchange (including  $Z$ - $Z'$  mixing and  $Z_2$  exchange) [16]. The corresponding expressions for those cross sections can be obtained from (36) by replacing the specification  $\text{SM} \rightarrow \text{AGC}$  and  $Z'$ , respectively. Accordingly, the double beam polarization asymmetry has a common form for all those cases:

$$A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{\text{AGC}} = A_{\text{double}}^{Z'} = P_1 P_2 = 0.48, \quad (37)$$

where the numerical value corresponds to the product of the electron and positron degrees of polarization:  $P_1 = 0.8$ ,  $P_2 = 0.6$ . Eq. (37) demonstrates that  $A_{\text{double}}^{\text{SM}}$ ,  $A_{\text{double}}^{\text{AGC}}$  and  $A_{\text{double}}^{Z'}$  are indistinguishable for any values of NP parameters, AGC or  $Z'$  mass and strength of  $Z$ - $Z'$  mixing, i.e.  $\Delta A_{\text{double}} = A_{\text{double}}^{\text{AGC}} - A_{\text{double}}^{\text{SM}} = A_{\text{double}}^{Z'} - A_{\text{double}}^{\text{SM}} = 0$ .

On the contrary, the heavy neutral lepton  $N$ -exchange in the  $t$ -channel will induce non-vanishing contributions to  $\sigma^{LL}$  and  $\sigma^{RR}$ , and thus force  $A_{\text{double}}$  to a smaller value,  $\Delta A_{\text{double}} = A_{\text{double}}^N - A_{\text{double}}^{\text{SM}} \propto -P_1 P_2 r_N^2 (G_L^{Ne} G_R^{Ne})^2 < 0$  irrespectively of the simultaneous lepton and  $Z$ - $Z'$  mixing contributions to  $\sigma^{RL}$  and  $\sigma^{LR}$ . A value of  $A_{\text{double}}$  below  $P_1 P_2$  can provide a signature of heavy neutral lepton  $N$ -exchange in the process (1). All those features in the  $A_{\text{double}}$  behavior are shown in Fig. 5, where we consider unpolarized  $W$ s.

The identification reach (ID) on the plane of heavy lepton coupling  $((G_L^{Ne})^2, (G_R^{Ne})^2)$  (at 95% C.L.) for various lepton masses  $m_N$  plotted in Fig. 6 is obtained from conventional  $\chi^2$  analysis with  $A_{\text{double}}$ . In that case the  $\chi^2$  function is constructed as  $\chi^2 = (\Delta A_{\text{double}} / \delta A_{\text{double}})^2$  where  $\delta A_{\text{double}}$  is the expected experimental uncertainty accounting for both statistical and systematic components. Note that discovery is possible in the green and yellow regions, whereas identification is only possible in the green region. The hyperbola-like limit of the identification reach is due to the appearance of a product of the squared couplings  $(G_L^{Ne})^2$  and  $(G_R^{Ne})^2$  in the deviation from the SM cross section, given by Eq. (28).

It should be stressed that the identification reach is independent of the  $Z'$  model assumed, whereas the discovery reach is not. In fact, in the lower left corner of these figures, we show how the discovery reach gets modified if we allow for  $Z$ - $Z'$  mixing within the  $Z'_\chi$  model (cf. Fig. 4).



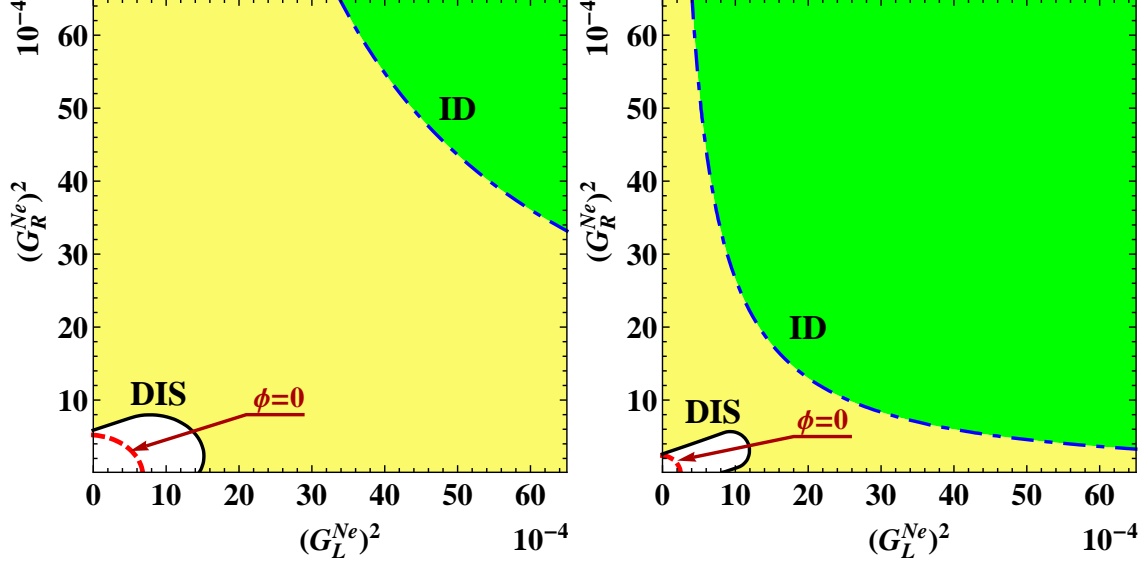


FIG. 6: Left panel: discovery (DIS) and identification (ID) reaches at 95% CL on the heavy neutral lepton coupling plane  $((G_L^{Ne})^2, (G_R^{Ne})^2)$ , obtained from a combined analysis of polarized differential cross sections  $d\sigma(W_L^+ W_L^-)/dz$  at different sets of polarization,  $P_L^- = \pm 0.8$ ,  $P_L^+ = \mp 0.6$ , and exploiting the double polarization asymmetry. Furthermore,  $m_N = 0.3$  TeV,  $\sqrt{s} = 0.5$  TeV and  $\mathcal{L}_{\text{int}} = 1$  ab $^{-1}$ . Right panel: similar, with  $\sqrt{s} = 1.0$  TeV and for  $m_N = 0.6$  TeV. The dashed curves labelled “ $\phi = 0$ ” refer to the case of no  $Z$ - $Z'$  mixing, whereas the outer contour labelled “DIS” refer to the minimum discovery reach in the presence of mixing.

## VI. DISCOVERY AND IDENTIFICATION REACH AT $\sqrt{s} = 350$ GEV

In view of the possibility of a staged ILC construction, we would like to comment on the possibility of obtaining bounds on heavy neutral leptons at 350 GeV. As illustrated in Fig. 7, polarized beams would already at this low energy allow to place a limit on possible  $NWe$  couplings, in particular at low masses  $m_N$ . In this figure we explore masses beyond the corresponding kinematical reach. Even at this rather low energy there is already sensitivity to discover heavy lepton couplings in the range of  $G^2 \sim 10^{-3}$  for low masses and up to  $G^2 \sim 5 \times 10^{-4}$  for heavy masses  $m_N$  and with an assumed integrated luminosity of 500 fb $^{-1}$ . It is seen that one can identify heavy-lepton-mixing effects for masses up to  $m_N \sim 400$  GeV.

Discovery is seen to become more sensitive at higher masses, since the effect is approximately proportional to  $1 - r_N$ , whereas for identification the sensitivity is governed by  $r_N$ , and thus becomes less efficient at higher masses. For higher beam energy, both sensitivities

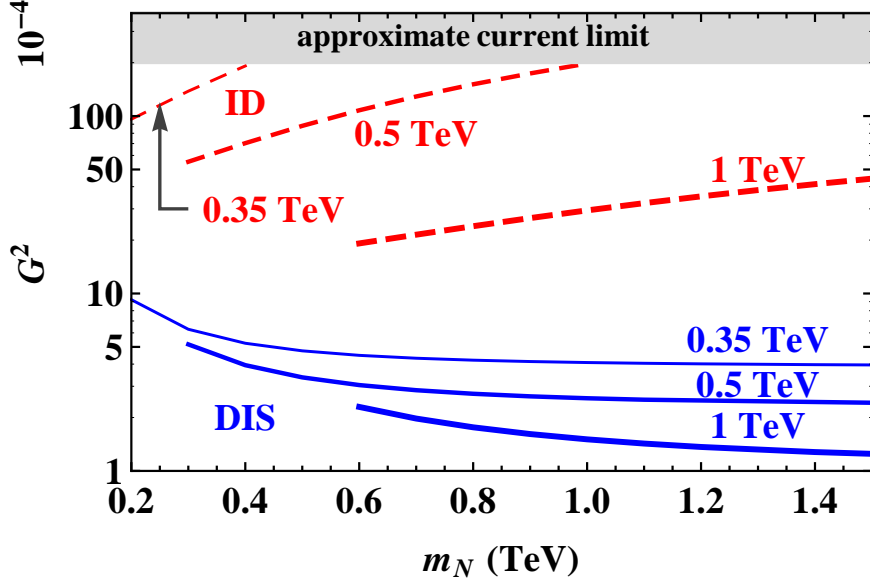


FIG. 7: Discovery (DIS) and identification (ID) reach on  $G^2 \equiv (G_L^{Ne})^2 = (G_R^{Ne})^2$ . The low-energy case (350 GeV) is compared with the nominal energy cases of 500 GeV and 1 TeV, all at an assumed integrated luminosity of  $500 \text{ fb}^{-1}$ . The approximate current limit on these couplings is indicated as a grey band.

improve.

## VII. CONCLUDING REMARKS

In this note we have studied the process  $e^+e^- \rightarrow W^+W^-$  and seen how to uniquely identify the indirect (propagator and exotic-lepton mixing) effects of a heavy neutral lepton exchange in the  $t$ -channel. Discovery of new physics, meaning exclusion of the Standard Model, does not depend on having both initial beams polarized, but the sensitivity is improved with beam polarization. Such “discovery” could be due to the existence of a  $Z'$ , anomalous gauge couplings, or the effect of a heavy neutral lepton. The potential of the ILC to discover heavy lepton effects depends on the possible presence of a  $Z'$  contribution, and is vastly improved if one is able to determine the polarization of the produced  $W$ s.

Identification of such new physics effect as being due to a heavy neutral lepton exchange, as opposed to a  $Z'$  or AGC can be achieved via the determination of a double polarization asymmetry. This identification of heavy-lepton admixture is independent of the strength of any  $Z$ - $Z'$  mixing, as well as the  $Z'$  model, but requires having both initial beams polarized.

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